

Even Decomposition of Some Special Class of Graphs

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Abstract

E.Ebin Raja Merly introduced the concept of Even Decomposition of a connected graph and Even Star Decomposition (ESD) of complete graph [4]. The Arithmetic Decomposition of G is said to be Even Decomposition (ED) of G if $\{G_1, G_2, G_3, \dots, G_n\}$ of subgraphs of G such that every edge of G is exactly once in G_i , each G_i is connected and $|E(G_i)| = 2i$, $1 \leq i \leq n$ [4]. In this paper we study the Even Decomposition (ED) of some special class of graphs namely $C_n \circledast K_1$ and $K_{1,n}^+$

Keywords

Continuous Monotonic Decomposition, Arithmetic Decomposition and Even Decomposition.

AMS Subject Classification: 05C70

Introduction

In this paper, we consider simple undirected graph without loops or multiple edges. The concept of Continuous Monotonic Decomposition was introduced by N.Gnana Dhas and J.Paulraj Joseph [1]. E.Ebin Raja Merly introduced the concept of Even Decomposition of a connected graph and investigate their variations [5]. V.Brishni, V.Maheswari and A.Nagarajan published the concept of Even Decomposition of Graphs [2]. From this paper, we continued the study of Even Decomposition (ED) of special class of graphs namely $C_n \circledast K_1$ and $K_{1,n}^+$

1.Preliminaries

Definition 1.1

Let $G = (V, E)$ be a simple graph of order p and size q . If $G_1, G_2, G_3, \dots, G_n$ are connected subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup E(G_3) \dots \cup E(G_n)$, then $(G_1, G_2, G_3, \dots, G_n)$ is said to be a decomposition of G .

Definition 1.2.

A decomposition $(G_1, G_2, G_3, \dots, G_n)$ of G is said to be continuous monotonic decomposition (CMD) if each G_i is connected and $|E(G_i)| = i$, for each $i = 1, 2, 3, \dots, n$.

Definition 1.3:

A decomposition $(G_1, G_2, G_3, \dots, G_n)$ of G is said to be a Linear decomposition or Arithmetic Decomposition if each G_i is connected and $|E(G_i)| = a+(i-1)d$, for all $i = 1, 2, 3, \dots, n$ and $a, d \in \mathbb{Z}^+$.

Definition 1.4

The corona $G_1 \circledast G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2 .

Definition 1.5

The graph $C_n \circledast K_1$ is called a Crown.

Definition 1.6

G^+ is a graph obtained from G by attaching a pendent edge to each of the pendent vertices of G .

Definition 1.7

The Arithmetic Decomposition of G is said to be Even Decomposition (ED) of G only when $a = 2$ and $d = 2$.

Lemma 1.8 [2]

Let $m \equiv 0 \pmod{2}$. The set $\{2, 4, 6, \dots, 2m\}$ can be partitioned into two sets S_1 and S_2 such that

$$\sum_{x \in S_1} x = \sum_{y \in S_2} y = n$$

Here $m(m+1) = 2n$.

Lemma 1.9 [2]

Let $m+1 \equiv 0 \pmod{2}$. The set $\{2, 4, 6, \dots, 2m\}$ can be partitioned into two sets S_1 and S_2 such that

$$\sum_{x \in S_1} x = \sum_{y \in S_2} y = n$$

Here $m(m+1) = 2n$.

2. Even Decomposition (ED) of $C_n \circledast K_1$ and $K_{1,n}^+$ **Theorem 2.1**

For any integer n , $C_n \circledast K_1$ has an ED $\{H_2, H_4, \dots, H_{2m}\}$ if and only if there exists an integer m satisfying the following properties:

- (i) $m = 2k$ or $2k - 1$ ($k \geq 1, k \in \mathbb{A}$)
- (ii) $m(m+1) = 2n$

Proof

Let $G = C_n \circledast K_1$. Then, $|E(G)| = 2n$. Assume $G = C_n \circledast K_1$ has a ED $\{H_2, H_4, \dots, H_{2m}\}$

Then $|E(H_2)| + |E(H_4)| + \dots + |E(H_{2m})| = |E(G)| \Rightarrow 2 + 4 + \dots + 2m = 2n$

$$2(1 + 2 + \dots + m) = 2n$$

$$2 \left(\frac{m(m+1)}{2} \right) = 2n$$

$$m(m+1) = 2n$$

$$m(m+1) \equiv 0 \pmod{2}$$

$$m(m+1) = 2k, k \geq 1, k \in \mathbb{A}$$

Hence $m = 2k$ or $m = 2k - 1$ ($k \geq 1, k \in \mathbb{A}$).

Conversely, assume $m(m+1) \equiv 0 \pmod{2}$.

Let $G = C_n \circledast K_1$. Let $V(C_n \circledast K_1) = \{u_1, u_2, u_3, \dots, u_n; v_1, v_2, v_3, \dots, v_n\}$ and $|E(G)| = 2n$.

Define

$$T_1 = \{(u_i, u_{i+1}): 1 \leq i \leq \frac{n}{2}\} \cup \{(u_i, v_i): 1 \leq i \leq \frac{n}{2}\} \text{ and}$$

$$T_2 = \{(u_i, u_{i+1}): \frac{n}{2} + 1 \leq i \leq n - 1\} \cup \{(u_i, v_i): \frac{n}{2} + 1 \leq i \leq n\} \cup \{(u_n, u_1)\}.$$

Here $|T_1| = n$ and $|T_2| = n$. Also, $|T_1| + |T_2| = 2 + 4 + \dots + 2m = m(m + 1)$. By lemma 1.10 and lemma 1.11, $\{2, 4, 6, \dots, 2m\} = S_1 \cup S_2$ where

$$\sum_{x \in S_1} x = n \text{ and } \sum_{y \in S_2} y = n$$

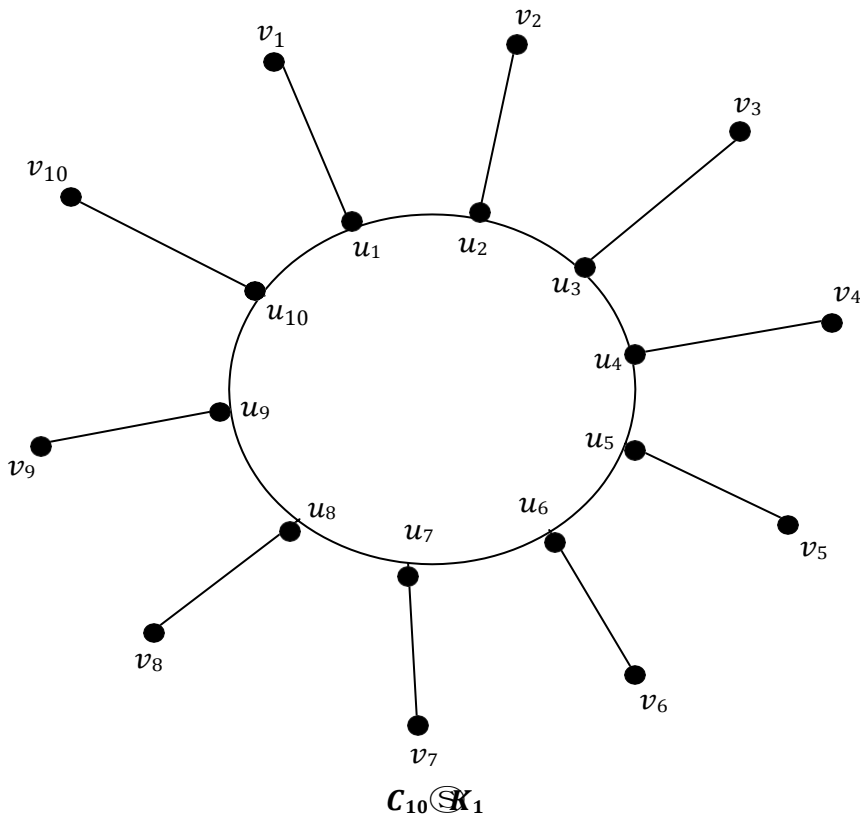
Decompose T_1 and T_2 into trees $\{H_i\}$ as follows

$$T_1 = \bigcup_{i \in S_1} H_i \text{ and } T_2 = \bigcup_{i \in S_2} H_i$$

Also, $|E(H_i)| = i, 1 \leq i \leq 2m$. Clearly $\{H_2, H_4, \dots, H_{2m}\}$ form an ED of C_n^+

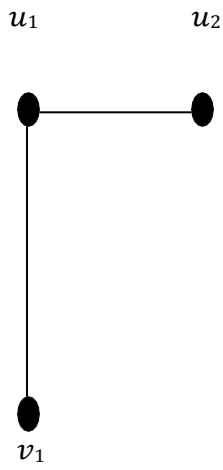
Illustration 2.2

As an illustration, let us decompose $C_{10} \circledast K_1$. Let $V(C_{10}) = \{u_1, u_2, \dots, u_{10}; v_1, v_2, v_3, \dots, v_{10}\}$. $C_{10} \circledast K_1$ is given below.

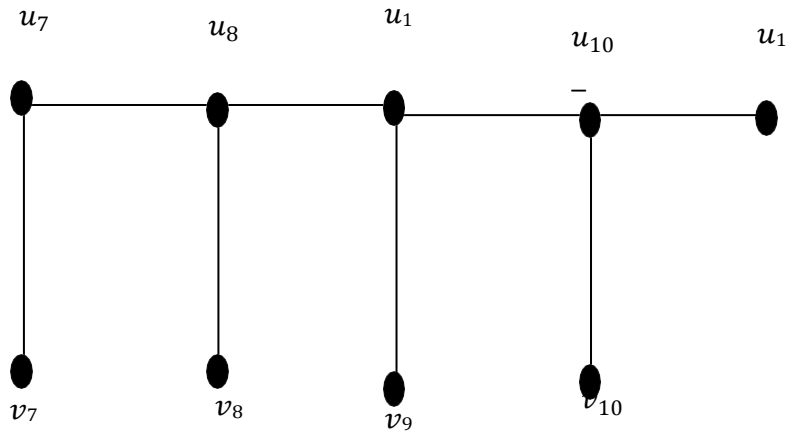


Here $|E(C_{10} \circledast K_1)| = 20$ and $m = 4$. $T_1 = \{(u_i, u_{i+1}): 1 \leq i \leq 5\} \cup \{(u_i, v_i): 1 \leq i \leq 5\}$ and $T_2 = \{(u_i, u_{i+1}): 6 \leq i \leq 9\} \cup \{(u_i, v_i): 6 \leq i \leq 10\} \cup \{(u_{10}, u_1)\}$. $|T_1| = |T_2| = 10$. Hence $|T_1| + |T_2| = 20 = 2 + 4 + 6 + 8 = 4(4 + 1)$. $S_1 = \{2, 8\}$ and $S_2 = \{4, 6\}$. T_1 is decomposed as $T_1 = H_2 \cup H_8$. T_2 is decomposed as $T_2 = H_4 \cup H_6$. Thus $\{H_2, H_4, H_6, H_8\}$ forms an ED of $C_{10} \circledast K_1$

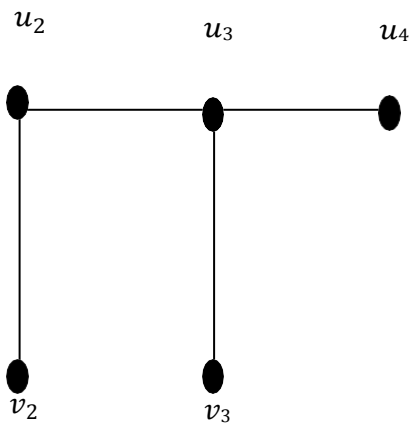
ED of $C_{10} \circledast K_1$



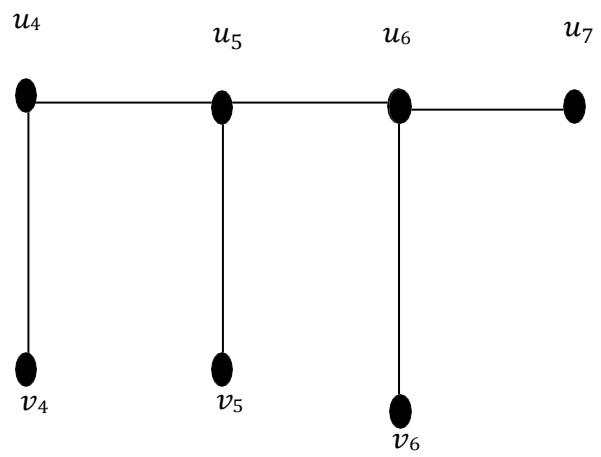
H_2



H_8



H_4



H_6

Theorem 2.3

For any integer n , $K_{1,n}^+$ has an ED $\{H_2, H_4, \dots, H_{2m}\}$ if and only if there exists an integer m satisfying the following properties:

- (i) $m = 2k$ or $2k - 1$ ($k \geq 1, k \in \mathbb{A}$)
- (ii) $m(m + 1) = 2n$

Proof

Let $G = K_{1,n}^+$. Then, $|E(G)| = 2n$. Assume $G = K_{1,n}^+$ has a ED $\{H_2, H_4, \dots, H_{2m}\}$

Then $|E(H_2)| + |E(H_4)| + \dots + |E(H_{2m})| = |E(G)| \Rightarrow 2 + 4 + \dots + 2m = 2n$

$$2(1 + 2 + \dots + m) = 2n$$

$$2 \left(\frac{m(m + 1)}{2} \right) = 2n$$

$$m(m + 1) = 2n$$

$$m(m + 1) \equiv 0 \pmod{2}$$

$$m(m + 1) = 2k, k \geq 1, k \in \mathbb{A}$$

Hence $m = 2k$ or $m = 2k - 1$ ($k \geq 1, k \in \mathbb{A}$).

Conversely, assume $m(m + 1) \equiv 0 \pmod{2}$.

Let $G = K_{1,n}^+$. Let $V(K_{1,n}^+) = \{u, u_1, u_2, u_3, \dots, u_n; v_1, v_2, v_3, \dots, v_n\}$ and $|E(G)| = 2n$.

Define

$$T_1 = \{(u, u_i): 1 \leq i \leq \frac{n}{2}\} \cup \{(u_i, v_i): 1 \leq i \leq \frac{n}{2}\} \text{ and}$$

$$T_2 = \{(u, u_i): \frac{n}{2} + 1 \leq i \leq n\} \cup \{(u_i, v_i): \frac{n}{2} + 1 \leq i \leq n\}.$$

Here $|T_1| = n$ and $|T_2| = n$. Also, $|T_1| + |T_2| = 2 + 4 + \dots + 2m = m(m + 1)$. By lemma 2.3 and lemma 2.4, $\{2, 4, 6, \dots, 2m\} = S_1 \cup S_2$ where

$$\sum_{x \in S_1} x = n \text{ and } \sum_{y \in S_2} y = n$$

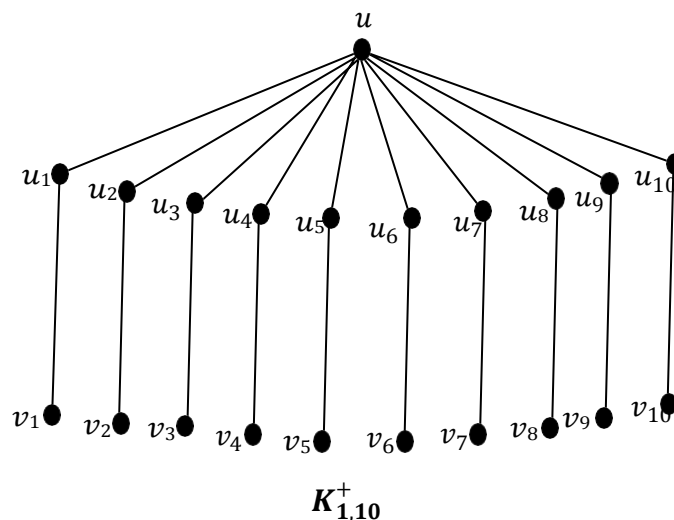
Decompose T_1 and T_2 into trees $\{H_i\}$ as follows

$$T_1 = \bigcup_{i \in S_1} H_i \text{ and } T_2 = \bigcup_{i \in S_2} H_i$$

Also, $|E(H_i)| = i, 1 \leq i \leq 2m$. Clearly $\{H_2, H_4, \dots, H_{2m}\}$ form an ED of $K_{1,n}^+$.

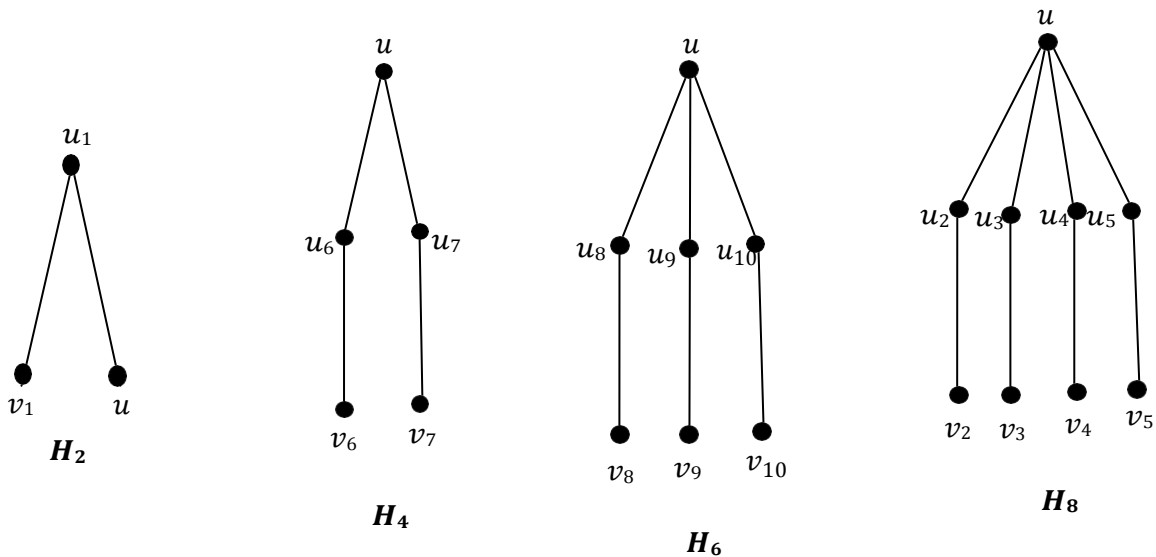
Illustration 2.4

As an illustration, let us decompose $K_{1,10}^+$. Let $V(K_{1,10}^+) = \{u, u_1, u_2, u_3, \dots, u_{10}; v_1, v_2, v_3, \dots, v_{10}\}$.



Here $|E(G)| = 20$ and $m = 4$. $T_1 = \{(u, u_i) : 1 \leq i \leq 5\} \cup \{(u_i, v_i) : 1 \leq i \leq 5\}$ and $T_2 = \{(u, u_i) : 6 \leq i \leq 10\} \cup \{(u_i, v_i) : 6 \leq i \leq 10\}$. $|T_1| = |T_2| = 10$. Hence $|T_1| + |T_2| = 20 = 2 + 4 + 6 + 8 = 4(4 + 1)$. $S_1 = \{2, 8\}$ and $S_2 = \{4, 6\}$. T_1 is decomposed as $T_1 = H_2 \cup H_8$. T_2 is decomposed as $T_2 = H_4 \cup H_6$. Thus $\{H_2, H_4, H_6, H_8\}$ forms an ED of $K_{1,10}^+$

ED of $K_{1,10}^+$



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